

Quark–antiquark potentials from a scalar field in $SU(2)$ YM

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Abstract. A generalized Dick model with a potential term is discussed. The solution originating from a static, point-like, color source is found to have a confining part. A comparison with a wide spectrum of phenomenological quark–antiquark potentials is presented.

1 The model

It is believed that the non-relativistic potential model can quite well describe the physics of heavy quarks. It is possible to obtain the whole spectrum of the masses of the quark and antiquark pairs in the quarkonium system, using only the potential $U(r)$. Here r denotes the distance between the quarks. Unfortunately, there are many different forms of the potential in the literature (see e.g. [1, 2]); nevertheless, up to now the final form has not been fixed. However, it was shown [3] that the most probable potential, in the bottomonia region, is

$$U_{\text{MZ}}(r) = C_1 \left(\sqrt{r} - \frac{C_2}{r} \right), \quad (1)$$

where $C_1 \simeq 0.71 \text{ GeV}^{1/2}$ and $C_2 \simeq 0.46 \text{ GeV}^{3/2}$. In the present paper we construct a Lorentz invariant, effective action which provides this potential.

The confining part of the potential, i.e. the part which is divergent in spatial infinity, can be obtained from the $SU(2)$ Yang-Mills theory coupled to the scalar field [4–6]. In particular, in the generalized Dick model [7], there is a sector where the confining potential has the same form as that given by (1). The main defect of the generalized Dick model (from the confinement point of view) is the simultaneous existence of finite energy solutions of the Coulomb problem. In order to preserve only the confining sector one has to add an additional potential term for the scalar field. This potential should have a unique minimum at $\phi = 0$ [7]. On the other hand, the Motyka–Zalewski potential (1) contains also the well-known Coulomb part. Because of this, our effective action should, in the limit of a strong scalar field, reduce to the simple YM theory. The effective action discussed below can also be regarded as a version of the $SU(2)$ color dielectric model with a special choice of the color dielectric function and scalar potential (see e.g. [8]).

Let us consider a model which satisfies the conditions mentioned above:

$$S = \int d^4x \left[-\frac{1}{4} \frac{\left(\frac{\phi}{\Lambda}\right)^{8\delta}}{1 + \left(\frac{\phi}{\Lambda}\right)^{8\delta}} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \alpha \phi^4 \left(\frac{\phi}{\Lambda}\right)^{8\delta} \right], \quad (2)$$

where $\delta > 0$, Λ is a dimensional constant, whereas α is a dimensionless one, and $F_{\mu\nu}^a$ is defined in the standard manner. We would like to stress that the potential for the scalar field has been chosen to give an analytic solution of the Coulomb problem.

The field equations for (2) have the following form:

$$D_\mu \left(\frac{\left(\frac{\phi}{\Lambda}\right)^{8\delta}}{1 + \left(\frac{\phi}{\Lambda}\right)^{8\delta}} F^{a\mu\nu} \right) = j^{a\nu}, \quad (3)$$

$$\partial_\mu \partial^\mu \phi = -2\delta F_{\mu\nu}^a F^{a\mu\nu} \frac{\phi^{8\delta-1}}{\Lambda^{8\delta} \left(1 + \frac{\phi^{8\delta}}{\Lambda^{8\delta}}\right)^2} - \alpha(4 + 8\delta) \frac{\phi^{8\delta+3}}{\Lambda^{8\delta}}, \quad (4)$$

where $j^{a\mu}$ is the external color current.

2 Solutions

Let us consider a static, point-like color source:

$$j^{a\mu} = 4\pi q \delta(r) \delta^{a3} \delta^{\mu 0}, \quad (5)$$

Without loss of generality the source can be taken to be Abelian. One can also consider a non-Abelian source, for example: $j^{a\mu} = 4\pi q \delta(r) C^a \delta^{\mu 0}$, where C^a is the expectation value of the $su(N_c)$ generator for a normalized spinor

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in the color space [4]. However, on account of the fact that the results for these two cases are very similar we will analyze only the Abelian source. The pertinent equations of motion read

$$\left[r^2 \frac{\left(\frac{\phi}{\Lambda}\right)^{8\delta}}{1 + \left(\frac{\phi}{\Lambda}\right)^{8\delta}} E \right]' = 4\pi q \delta(r), \quad (6)$$

$$\nabla_r^2 \phi = -4\delta E^2 \frac{\phi^{8\delta-1}}{\Lambda^{8\delta} \left(1 + \frac{\phi^{8\delta}}{\Lambda^{8\delta}}\right)^2} + \alpha(4 + 8\delta) \frac{\phi^{8\delta+3}}{\Lambda^{8\delta}}, \quad (7)$$

where $\mathbf{E}^a = E\delta^{3a}\hat{r}$ is the electric field defined in the standard way. These equations possess the following solutions:

$$\phi(r) = \Lambda A \left(\frac{1}{\Lambda r}\right)^{1/(1+4\delta)}, \quad (8)$$

$$\mathbf{E}(r) = \left[\frac{q}{r^2} + A^{-8\delta} q \Lambda^2 \left(\frac{1}{\Lambda r}\right)^{2/(1+4\delta)} \right] \hat{r}, \quad (9)$$

where

$$A = \left[\frac{-4\delta + \sqrt{4\delta(1 + 4\alpha(1 + 2\delta)(1 + 4\delta)^4)}}{8\alpha(1 + 2\delta)(1 + 4\delta)^2} \right]^{1/(2(1+4\delta))}.$$

This number is positive for sufficiently large values of α . The energy of the solutions is divergent, not only in the small r limit, but also in the long range limit, $r \rightarrow \infty$. In this sense, confinement emerges.

For $\delta > 1/4$ the color electric potential has the form

$$V(r) = -\frac{q}{r} + \frac{4\delta + 1}{4\delta - 1} A^{-8\delta} q \Lambda^{4\delta/(4\delta+1)} \cdot r^{(4\delta-1)/(4\delta+1)}. \quad (10)$$

Let us assume that the color source is a heavy quark. Therefore, the potential seen by an (anti)quark has the following form:

$$U(r) = -\frac{q^2}{r} + \frac{4\delta + 1}{4\delta - 1} A^{-8\delta} q^2 \Lambda^{4\delta/(4\delta+1)} \cdot r^{(4\delta-1)/(4\delta+1)}. \quad (11)$$

A similar calculation can be done for the case $\delta = 1/4$. The result is

$$U(r) = -\frac{q^2}{r} + \Lambda A^{-8\delta} q^2 \ln \Lambda r. \quad (12)$$

For $\delta < 1/4$ the potential does not show a confinement-like behavior.

There are three general conditions which must be satisfied by a static potential. Namely, it cannot rise faster than linearly as a function of the distance r for $r \rightarrow \infty$ [9], it has to be a monotonically increasing function of r , and $U''(r) \leq 0$ [10]. Unfortunately, our potential (11) satisfies these conditions for all $\delta > 1/4$ and we cannot use them to constrain the parameter δ .

The potential term in (2) not only excludes single charge states from the physical sector of the theory (i.e.

there are no finite energy solutions of the Coulomb problem), but it also removes magnetic monopoles. Let us rewrite the equations of motion using the well-known magnetic monopole Ansatz for the gauge field:

$$A_i^a = \epsilon_{aik} \frac{x^k}{r^2} (g(r) - 1), \quad A_0^a = 0, \quad (13)$$

where $g(r)$ is a function of the radial coordinate only. We get

$$\left[\frac{\left(\frac{\phi}{\Lambda}\right)^{8\delta}}{1 + \left(\frac{\phi}{\Lambda}\right)^{8\delta}} g' \right]' + \frac{\left(\frac{\phi}{\Lambda}\right)^{8\delta}}{1 + \left(\frac{\phi}{\Lambda}\right)^{8\delta}} \frac{g}{r^2} (1 - g^2) = 0, \quad (14)$$

$$\begin{aligned} \frac{1}{r^2} (r^2 \phi')' &= 4\delta \frac{\phi^{8\delta-1}}{\Lambda^{8\delta} \left(1 + \frac{\phi^{8\delta}}{\Lambda^{8\delta}}\right)^2} \left[\frac{2g'^2}{r^2} + \frac{(g^2 - 1)^2}{r^4} \right] \\ &+ \alpha(4 + 8\delta) \frac{\phi^{8\delta+3}}{\Lambda^{8\delta}}. \end{aligned} \quad (15)$$

The above set of equations possesses the unique but trivial finite energy solution $\phi = 0$. The finite energy monopoles observed in [7] do not appear due to the potential term.

3 Conclusions

We can compare the quark–antiquark potential derived from model (2) with the phenomenological confining Motyka–Zalewski potential. It is immediately seen that they become identical if we set $\delta = 3/4$.

However, one can fit the model to another phenomenological potential, which has been successfully applied to calculate quarkonium energy levels, namely to the Cornell potential $U_C(r) = -(a/r) + br$, where a, b are non-negative constants [1]. Our potential has the same form in the limit $\delta \rightarrow \infty$, but it is not feasible to implement this limit on the lagrangian level. So, strictly speaking, our model does not supply the linear divergence of the confining potential, but this can be achieved with arbitrary accuracy by taking sufficiently large values of δ . That is the main difference between the Dick model and the model presented here.

Contrary to the model considered in [7], there exists no solution which is non-singular in spatial infinity. It was gained by adding the potential term for the scalar field. There are no magnetic monopoles, either. It is worth stressing that in the model confinement and disappearance of the magnetic monopoles occur simultaneously.

There are, at least, two directions in which the present work might be continued. Firstly, theoretical restrictions for the parameter δ as well as for the particular form of the potential term in the action (2) are needed. Due to the fact that our action belongs to a wide family of color dielectric actions we believe that these problems can be solved using the lattice color dielectric methods. In fact, much work was done in the past to derive the lattice color dielectric

model from the lattice QCD (see e.g. [11–13]). For example in [12] the effective scalar potential for $\delta = 1/2$ has been computed. One can also use the detailed, lattice study of the flux-tube profile, which was done in the last few years (see e.g. [14–16]) and compare it with the predictions of our model to fix the δ parameter.

Secondly, because finite energy solutions of the Coulomb problem appear for potentials which have a unique minimum for $\phi \neq 0$, the model can be used to study the confinement–deconfinement phase transition.

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